NRCC WORKING GROUP ON RIVER ICE JAMS

GUIDELINES FOR EXTRACTION OF ICE-BREAKUP DATA FROM HYDROMETRIC STATION RECORDS

by

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FOREWORD

This report has been prepared for the NRCC Working Group on River Ice Jams and addresses the Group's Task 2: "To develop guidelines for analyzing pertinent data that are already available." The membership of the Working Group is as follows:

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1.0 INTRODUCTION

A major consequence of ice cover formation in rivers is the jamming that occurs during spring breakup of the ice cover. Because of their large aggregate thickness and hydraulic resistance relative to those of sheet ice, jams cause unusually high water stages. This has repercussions in many operational and design problems such as flooding and associated flood risk assessments; overturning moment applied to river structures by moving ice floes; bed scour due to surges from ice jam releases; and many others.

The available technology to deal with problems related to ice jams is very limited. This reflects the great complexity of ice jamming processes and Kennedy's (1975) observation that "...an ice-jammed river is among the most deranged of hydraulic phenomena" is still very much valid today. This complexity of ice jamming phenomena imposes a serious limitation on the type of pertinent research that can be performed, that is, it renders reproduction of all or even most of the pertinent factors in the laboratory very difficult. As a result, laboratory experimentation on ice jams must be limited to the study of specific, isolated aspects in cases where field observation has helped identify the basic processes involved. As an example, one could cite the case of ice jams formed entirely by submergence of ice blocks. In this instance, laboratory tests can furnish useful data on the thickness and thence the water stage of such jams. However, this knowledge only addresses a very small part of the entire problem which briefly can be stated as follows: "In a given ice covered reach, increased runoff and mild weather are anticipated in the next few days. Forecast whether and when ice breakup will be initiated; forecast locations, durations and peak stages of any ice jams that might occur; forecast when the breakup will be completed."

Clearly, the study of river ice jams must be based, to a large degree, on field observation and data. This perception has recently become a general one among hydraulic engineers so there is good reason to expect increased monitoring activities in the future. However,

development of an adequate base of field case studies will require many years after a national-scale observation program has been initiated.

In the meantime, it would be desirable to consider other sources of field data that might already be available. One such source is the hydrometric gauge data obtained routinely by Water Survey of Canada (WSC) and by other agencies throughout a national gauge network. This source is imperfect (ice effects are only documented to improve discharge estimates) but has two attractive features: it is available readily at little cost; and it adequately covers the Canadian ranges of stream and weather types.

Earlier work (Shulyakovskii 1963; Beltaos 1981, 1982; and Beltaos and Lane 1982) has shown that this approach has good potential for development of needed forecasting methods. Achievement of this potential, however, requires analysis, interpretation and comparison of data from many gauge sites. This report provides guidelines for extracting ice-related information from the hydrometric records in the hope that others may be encouraged to carry out such studies so as to accelerate the formation of an adequate data base. Examples of utilizing the results of the analysis are included for illustration purposes. As a by-product, instances are identified where minor additions to gauge operation procedures would substantially improve the utility of the records.

2.0 RIVER ICE BREAKUP

When an ice-covered river basin is subjected to mild weather, two major processes generally begin:

i) Increased runoff by either snowmelt or rainfall, or both.

ii) Increased heat input to the ice cover.

The former process results in increased uplift and frictional forces applied to the ice cover and in increased water stage which, in turn,

reduces the support provided to the ice cover by the channel banks and provides increased channel width for movement of the ice cover. Net heat input to the ice cover results in reduced dimensions and strength. It follows that during the mild weather spell, the forces applied on the ice cover increase while the cover's ability to resist these forces decreases. If the mild weather lasts for a sufficient time, the ice cover begins to break up which is often followed by formation of large ice jams, major ice runs and eventual clearance of the ice from the reach of interest. This general description of the breakup process includes two extreme cases, i.e., the "premature" and "overmature" breakup (Deslauriers 1968). Premature breakup occurs under conditions of intense runoff with little, if any, prior deterioration of the ice cover. Clearly, this type of event has the greatest damage potential, other things being equal. On the other hand, conditions of slow or no runoff with intense ice deterioration lead to overmature breakup. This event is characterized by gradual ice disintegration and has minimal potential for damage.

The first question a forecaster might ask would be how to predict whether and when breakup will be initiated and, once initiated, how severe it is likely to be in terms of magnitude and duration of ice jam stages at various locations.

Concerning breakup initiation, pertinent literature often advocates use of the corresponding water stage, H_B (= height above an arbitrary datum, e.g., gauge height) as a convenient and meaningful index (Shulyakovskii, 1963; Gerard, 1979; Beltaos, 1981, 1982). From our earlier discussion, it would appear that H_B is indeed a desirable parameter because it reflects the ice driving forces as well as the water surface width available for ice movement. Moreover, the above noted literature suggests that, in a given reach, H_B depends on: the thickness of the ice just prior to breakup, h_i; the degree of ice strength reduction caused by thermal effects; and the stage during freeze up when a stable ice cover forms, H_F. The latter is a measure of the width of the ice cover and, excepting mature breakup events, has

to be exceeded before contact of the ice with channel boundaries is eliminated. As will be discussed later, approximate values of these parameters can be extracted from gauge records. As for ice strength, there is no direct information. The best that can be done at present is to use a meteorological index intended to reflect the effects of thermal deterioration.

With regard to the severity of breakup, one would ideally wish to predict the complete stage hydrograph during the breakup period, at any given location. This appears to be too ambitious a task at present; it is thought more practical to limit the goal of the study to forecasts of the maximum stage during breakup, H_m. This stage can easily be identified on gauge recorder charts and is usually caused by a nearby ice jam. Theoretical considerations and field data (Pariset et al, 1966; Beltaos, 1983a) have shown that the maximum stage that can be caused by an ice jam occurs when the jam has attained a condition of equilibrium and fully affects the site of interest. This equilibrium stage depends mainly on discharge as well as channel slope and width. During any one breakup period, H_m may or may not reach the equilibrium jam stage owing to one or more of the following reasons:

i) Ice jam is located far downstream of the gauge site. Even if this jam attains equilibrium, the gauge site will experience a fraction of the jam's effect on stage.

ii) Ice jam is located far upstream of the gauge site. The gauge site will again experience a fraction of the jam's effect on stage owing to attenuation effects during the jam's release.

iii) Unstable jam that releases prior to attaining equilibrium.

iv) Overbank flooding. Water and ice spread out onto the flood plain so that the jam's potential is dissipated. This case could be viewed as a particular instance of the unstable jam case.

Considering that a major factor contributing to ice jam formation is the original ice cover itself, it is reasonable to expect that not only

discharge but also the competence of this cover may influence the value of H_m (see also later discussion).

The preceding discussion has identified several parameters that can be determined from existing data and that can be expected to be related to important ice jam features.

3.0 DESCRIPTION OF EXISTING DATA

The following data are routinely obtained throughout the year at any one gauge site.

3.1 Stage Record

At automatic recording gauge sites, a continuous stage versus time chart is available. Our main interest is in the ice cover period which occasionally may contain intervals of missing records owing to malfunctions caused by the ice. This limitation may be too severe at some gauge sites to enable data analysis but there are many other sites where such malfunctions are minimal.

3.2 Local Observer's Reports

At many gauge sites, local observers are temporarily employed by the operating agency to provide brief descriptions of ice conditions at a specified frequency. This information is extremely useful, especially in cases where interpretation of the stage record is difficult.

3.3 Discharge Measurement Notes and Ice Thickness

During the ice cover period, discharge measurements are made periodically to enable calculation of daily values. Since conventional stage-discharge relations are not effective during this

time, a number of interpolative techniques are used to calculate the daily discharges. Additional information, such as flows at adjacent sites, precipitation, and temperature, is also used in the calculation.

These calculations may be in considerable error during periods of intermittent ice cover or periods of highly unsteady flow such as during the break-up period. Examination of the timing of discharge measurements with respect to the calculated discharge hydrograph during the ice cover and break-up periods, and examination of the measurement notes themselves, help to assess the reliability of the calculated daily discharges.

In addition, the discharge measurement notes can be a useful source of ice thickness data because the distance from the water surface to the bottom of the ice cover is recorded at each measurement hole. Under free-flotation conditions, this quantity represents about 92% of the total ice thickness. In reality, this relationship may not be valid owing to one or more of the following possibilities:

i) Significant bank support of the ice cover.

ii) Snow cover.

iii) Slush deposit under the solid ice layer. The notes would only show the distance from the water surface to the bottom of the slush layer.

iv) Unreported instances where "water surface" has been used nominally, i.e., substituted by a more convenient datum such as the top of a deep snow layer.

Concerning the submerged to total thickness ratio of a solid ice sheet, it is very difficult to select an accurate value without knowledge of local conditions. It is not uncommon for the water surface to come very near the top of a hole cut in an ice sheet and, sometimes, to rise

above it. Perhaps the best approach would be to agree on a value of the desired ratio and use it as a matter of convention. For example, a value of 0.96 or 0.97 would seem a fair compromise since it could not be in error by more than a few percent.

Clearly, future measurements of the distance from the water level to the top of the ice cover and the solid versus slush ice fractions would provide more reliable thickness values.

3.4 Meteorological Records

In addition to hydrometric records, meteorological information is needed at a nearby weather station to define an index for ice strength. Meteorological data may be found in the publication "Monthly Records" that is issued by Atmospheric Environment.

4.0 EXTRACTION OF INFORMATION RELATED TO BREAKUP

After the necessary raw data have been gathered, a number of pertinent parameters can be extracted, as outlined next.

4.1 Stable Freeze Up Stage (H_F)

A typical but not universal configuration of the daily average stage hydrograph near the start of the ice season is sketched in Figure 1. The solid line represents the actual stage whereas the broken line gives the "effective" stage (= stage that would have occurred had the flow been unaffected by ice). At a certain time, which may be termed the beginning of freeze up, the actual stage starts to rise while the effective stage continues to drop. Eventually, the actual stage attains a peak and then declines. This sequence reflects the dynamic nature of ice cover formation in rivers. With the onset of cold weather, frazil ice forms and is initially transported freely. The effect of this frazil ice on the water stage is small. As more and more

frazil is produced, it begins to agglomerate into slush and pancakes. Eventually, the ice transport is impeded somewhere downstream of the gauge (due to border ice growth or other constricting feature) and an ice cover begins to propagate upstream. The presence of the ice cover causes a local stage increase which eventually begins to be "felt" at the gauge site. The gauge height then increases until the time when the edge of the ice cover arrives at the gauge site. Subsequently, the gauge height decreases due to the combined effects of decreasing discharge and thermal smoothing of the underside of the cover. The peak stage (H_F) during this period is considered an important factor influencing the succeeding breakup because the width of the cover is approximately equal to the channel width at the stage H_F. To eliminate brief peaks during which there is little time for freezing, H_F is defined as a daily average value. It should be recognized that the above description may not apply in cases where there are severe flow and stage controls; where conditions are such that a complete ice cover does not form; where the discharge is decreasing so rapidly as to suppress the occurrence of a peak on the daily stage hydrograph; or where a precipitous stage drop occurs after the time of the peak so that the stable ice cover width cannot be represented by this peak*. Because these and possibly other uncertainties are not generally known beforehand, it is advisable to determine H_F in conjunction with reference to prevailing temperature and precipitation conditions as well as local observers' notes.

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* For example, in some of the MacKenzie River delta channels, the flow is cut off by a drop of the main river stage, shortly after freeze up. Consequently, the stable ice cover in these channels forms at a much lower stage than the original freeze up peak (T. Prowse, personal communication).

An example of an actual freeze up stage record is given in Fig. 2 where the wavy appearance is caused by upstream regulation. The record shows that ice effects began to be experienced at the gauge site in the early morning of November 15, 1972. This was followed by a sharp rise of 1.6 m by evening and a gradual decline afterwards. The calculated daily average stage hydrograph is, of course, much smoother than the instantaneous and shows a peak of 4.29 m on November 16. This value is thus taken as H_F which is also in agreement with the observer's report for November 16.

4.2 "Winter Peaks"

Occasionally, a brief thaw may occur during the winter period. If such a thaw causes significant runoff, the gauge record will show a peak which may or may not initiate breakup. In the latter case, the peak stage represents a lower limit for the stage required to initiate breakup at that time.

4.3 Stage at Initiation of Breakup (H_B)

Usually, when a thaw does lead to breakup of the ice cover, the stage begins to rise from its fairly steady winter value and shortly after exhibits spikes and peaks that can only be caused by breaking or broken ice effects (Fig. 3). A probable value of the stage at the initiation of breakup, H_B, may be fixed at the first significant spike. Unfortunately, this definition is not always objective or meaningful (see also later discussion)*. Only a probable range of H_B can then be determined, by considering: (a) the latest time for which it can be confidently assumed that there still was continuous ice cover;

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* Initiation of breakup is defined herein as the instant when a sustained movement of the ice cover begins. When the cover is set in motion, the resistance to flow is reduced and the stage should tend to drop, thus producing a spike on the stage hydrograph. Sometimes, however, the stage rise may be so steep as to suppress spike appearance. Only a slowdown in the rate of rise would then be evident.

and (b) the earliest time for which broken ice effects became evident on the stage hydrograph. An illustration of actual breakup conditions is given in Fig. 4. The water level is seen to rise during April 18, 1973, reaching a first peak of 4.42 m at 0600 on April 19. A sharp peak of 5.05 m occurred at about 1630 on April 19, followed by a quick drop of about 0.2 m and renewed rise. The second peak is a more likely value for H_B because ice movement is usually associated with much steeper falling limbs than that of the first peak. This is confirmed by the local observer's report for April 19 and thus H_B is fixed at 5.05 m.

However, identification of H_B may not always be as simple as indicated above, especially in cases where no local observations and reports are available. Particular difficulty would then be experienced in cases of:

i) Absence of spikes owing to very rapid increases in stage, caused by intense runoff or release of major ice jams upstream.

ii) "Misleading" spikes that may be caused by discharge reductions due to upstream jam formation or even disruption of the melt process.

iii) Overmature events where breakup can be initiated during constant or even dropping stage conditions. The ice cover would then be disintegrating gradually and our earlier definition of breakup initiation may not apply.

An example of overmature breakup is given in Fig. 5, showing the stage hydrograph for the 1982 breakup period in Peace River. Breakup was initiated near midnight of April 26 while the stage was generally decreasing (Fonstad 1982). The wording "breakup front" refers to the configuration sketched in the inset of Fig. 5: a relatively short accumulation of broken ice rests against the upstream edge of stationary sheet ice. Thermal and mechanical action causes the edge of the stationary ice to develop cracks while open leads form downstream. Eventually, the deterioration becomes so advanced that the broken ice is able to move into the leads where it comes to a temporary halt and the

process is repeated. The fact that the broken ice accumulation does not lengthen with time implies that melting is one of the governing factors. Breakup initiation in this case can be defined as the passage of the "front", but it would be difficult to identify it on a stage recorder chart without local observer's reports. Moreover, the importance of H_B as an index of breakup initiation would probably be limited to its effect on flow velocity under the ice which, in turn, influences the rate of heat transfer from the water.

Because of the relatively little importance of the overmature breakup to ice jamming, the remainder of this report will concentrate on non-overmature breakup events. Before closing this section, it is emphasized that the accuracy of H_B determinations would be greatly increased by availability of frequent local observer's reports during the breakup period (about 3-4 times a day); and by automatic recording of ice conditions via time-lapse photography or ice movement detection devices.

4.4 Maximum Breakup Stage (H_m)

This is the maximum stage reached during the breakup period and its determination is usually straightforward (see Figure 2). However, malfunctions caused by ice or other effects may cause problems. For example, Fig. 4 shows two "suspiciously" flat and equal peaks near the respective midnights of April 19 and 20. In the absence of additional information, these peaks cannot be used as representative of H_m. It would thus seem highly desirable to instruct the local observer to identify the high water mark during each breakup event. During a subsequent visit of the gauge by operating agency staff, the high water mark could be surveyed.

4.5 Effective Stage and Maximum Ice Effect on Stage (ΔH_m)

The ice effect on stage is the difference between the actual stage and the effective stage. The time of maximum ice effect can

usually be determined by simple inspection (Figure 3) and does not necessarily coincide with the time of H_m. Since the effective stage is obtained from discharge estimates as described in Section 3.3, ΔH_m values are subject to the same errors associated with discharge estimates during breakup. Examination of discharge measurement notes would enable subdivision of ΔH_m data into fair and uncertain ones. A more accurate indication of the ice effect on stage is the value obtained from a discharge measurement that happened to be performed during the breakup period. Such data should also be recorded for comparison purposes even though they do not generally represent the maximum ice effect, ΔH_m.

4.6 Ice Thickness (h_i)

Usually, a few ice thickness values will be available during any one winter season. These can be plotted versus time and an extrapolation can be made to the start of the mild weather spell that led to breakup. Where the winter season involves highly variable weather conditions it may be preferable to extrapolate using a more complex correlation, e.g., h_i versus accumulated degree-days of frost. Such procedures would generally give fair indications of h_i at the time breakup starts but ignore thickness reductions that may occur during the pre-breakup period (onset of mild weather spell to onset of breakup). This assumption is considered adequate for the present in view of (a) the crudeness of the other data involved; and (b) the partial accounting of this effect by introducing a meteorological index of heat input to the ice cover.

4.7 Meteorological Index of Ice Strength

Few data on ice strength at the time of breakup are available and the manner of ice strength reduction by thermal effects is not well understood at present (Frankenstein, 1961; Korzhavin, 1971; Butyagin,

1972; Ashton, 1983). In general, it is reasonable to expect that ice strength will decrease with increasing amounts of heat absorbed by the ice cover but there is no consensus on the most appropriate index for the latter.

A very simple and well known index is the accumulated degree-days of thaw, S_T (see for example, Williams, 1965; Bilello, 1980). It is clear, however, that S_T can only be satisfactory in cases where the time of year when breakup occurs and the number of "thawing" days do not vary appreciably. Otherwise, the important effect of solar radiation will not be considered. For example, a sunny day in April would be much more effective in weakening the ice than a cloudy day of the same average air temperature in January. It is evident that, to fully account for thermal effects on ice strength, several parameters are needed in addition to air temperature, e.g., short wave radiation, cloudiness, wind speed, water temperature, snow cover, ice composition, etc.

Unfortunately, not all of this information is usually available and even if it were, it would be impractical to attempt multiple correlations with so many parameters. Shulyakovskii (1963) suggested the use of a calculated value of heat input to the ice cover from the surface, thus ignoring heat transfer from the water since water temperature is, as a rule, unknown. A similar but somewhat simplified approach was suggested by Williams (1965). Bulatov (1972) outlined a method for computing ice strength based on theoretical and experimental correlations with radiation effects. However, Bulatov's paper was too general to permit application of his method by others. Ashton's (1983) analysis is similar to Bulatov's and shows that the main agent of deterioration is the penetrating solar radiation, once the ice has been warmed to 0°C. Additional radiation absorption causes melting at the grain boundaries with a resulting decrease in strength. However, Ashton's analysis cannot be applied to the data under consideration because information on snow cover, albedo and ice structure is lacking.

Evidently, only empirical indices of ice strength can be employed at present. Some of the simplest ones are accumulated degree-days of thaw, hours of sunshine and solar radiation but their simultaneous consideration would complicate the analysis. Shulyakovskii's (1963) single heat input parameter, $\sum q$, has the advantage of simplicity as well as a background of practical usage. However, there is no theoretical evidence that this parameter adequately accounts for the qualitatively different effects on ice strength of the various heat components involved. A possible means of resolving this question is to perform a few comparative studies, i.e., to compare the forecasting effectiveness of Σq versus that of other indices such as accumulated degree-days, hours of sunshine, solar radiation, etc.

5.0 DISCUSSION

At present no definite rules can be offered for interpreting data gathered according to the previous section because it is still too early to confidently adopt a general interpretation methodology.

A partial illustration of data interpretation is given in the Appendix where a case study on the Nashwaak River, New Brunswick, is described. This study (Beltaos and Lane 1982) utilizes some of Shulyakovskii's (1963) suggestions along with more recent understanding of ice jams. The main findings of the study* are broadly as follows:

i) Forecasting the time of breakup initiation. The corresponding stage H_B is a suitable index while the difference $(H_B - H_F)$ is related to h_i and a measure of thermal ice deterioration. For the latter, several indices have been used in the past but there is no concensus on which is the optimum. For the Nashwaak

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* and also of several similar case studies at other gauge sites (Beltaos, Unpublished Data). None of these studies includes overmature breakup events.

River study, Shulyakovskii's heat input parameter was used and gave fair correlations. The effects of channel geomorphology (fixed for a given site) are unknown at present; they can only be investigated by comparison of several site-specific case studies.

ii) Forecasting the severity of breakup. The peak breakup stage, H_m, is known to be influenced by discharge while preliminary information suggests that H_F, h_i and thermal deterioration may play a role. The effects of channel geomorphology are partly understood via the theory of equilibrium jams. At present, only upper bounds of, or potential, H_m values can be forecast. Actual H_m predictions are not possible.

iii) Frequency of H_m. This can be done simply by ranking H_m values and computing corresponding probabilities. Use of H_m', a value reduced by the stage of zero discharge, is convenient. Preliminary indications are that H_m' may adhere to the log-normal distribution which can be defined by specification of two parameters. Comparison of several case studies might result in correlations of these parameters with geomorphic and basin characteristics.

Earlier considerations have highlighted several instances where relatively modest increases in the effort associated with the operation of a gauge, would result in very important contributions toward development of forecasting methods, specifically:

- Accurate ice thickness values can be obtained by recording the distance of the water level to the top of the ice cover and the solid versus slush ice fractions where applicable.
- The accuracy of H_F and (more importantly) H_B determinations would be greatly enhanced by increasing the frequency of the local observer's reports to three or four times a day during freeze up and breakup. Ice movement detection devices, time-lapse photography or simply occasional photographs would also be helpful.

- At some locations, broken ice causes the gauge to malfunction so that H_m values are, as a rule, missing. Identification of the high water mark during breakup by the local observer would enable later survey and determination of H_m. In doing this, care should be exercised to ensure that H_m applies to ice-influenced flow conditions and not merely to open-water flow.

6.0 GUIDELINES

The procedures for performing a breakup case study are summarized in this section.

6.1 Selection of Gauge Site

The following criteria should be examined prior to selection:

i) A significant ice cover (thicker than about 10 cm) forms in the vicinity of the gauge location during most winters.

ii) The length of record is adequate, i.e., more than about 15 years.

iii) Water level recorder malfunctions during breakup are rare.

iv) River geometry has not been subjected to significant changes during the length of record. One indication regarding this aspect is the stability of the gauge rating curve.

v) Accessibility of gauge site and vicinity. It may eventually be desired by users to survey a few cross-sections and even perform in situ ice observations as a supplement to data interpretation.

vi) Availability of local observer's reports.

vii) Discharge measurements under ice-covered conditions are, as a rule, carried out. Associated ice thickness indications are representative of the solid ice layer, i.e., no major slush accumulations.

viii) Discharge estimates during breakup are relatively accurate owing to favourable conditions, e.g., timely measurements, upstream and downstream gauges where ice breakup is completed earlier, etc.

ix) Proximity of representative meteorological station(s).

x) Consideration should be given to study sites where snow run-off prediction models such as SSARR are already in place. Here a possibility exists to relate the defined snow conditions to the strength of the ice cover with respect to break-up.

6.2 Descriptive Information on Ice Regime

An overall qualitative description of the ice regime in the vicinity of the gauging station would be helpful in breakup data interpretation. Presentation of, or reference to, the following information is recommended.

i) River profiles and plan view of river obtained from 1/50000 maps, flood plain maps and/or other existing water resources reports on the river.

ii) Monthly and daily discharge characteristics of the river (mean, maximum and minimum of record).

iii) Statistical meteorologic conditions at the site described in Canadian and Provincial climatologic atlases.

iv) Satellite photos taken at regular intervals over winter. These photos are particularly useful for large rivers where formation and progression of the ice cover is relatively slow. The open water zones can be clearly distinguished from the ice covered portions of the river in the absence of cloud cover.

v) Interviews with local residents are optional but are highly recommended for more detailed studies.

6.3 Data Processing

For each season, the following items are needed:

i) Identify approximate period of ice cover. If there are more than one ice cover periods, identify and process each one separately.

ii) Identify and record time and value of H_F. Consult: recorder chart; daily average stage table; local observer's reports; and weather conditions.

iii) Inspect recorder chart after the time of H_F. Identify significant winter peaks and record their times and values.

iv) Roughly locate the breakup period on the chart, i.e., start of final rise of water level to end of ice effect. Draw the effective stage graph by joining with a smooth line daily average values plotted at noon of corresponding days. Effective stage can be found from estimated discharge via the gauge rating curve or table. Consult weather conditions and local observer's reports and make notes where appropriate. Identify and record times and values of H_B, H_m and stage corresponding to ΔH_m, along with applicable discharges. If H_B can only be identified as a range, record parameters pertaining to the limits of the range.

v) Examine applicable discharge measurement notes. Compute average nominal "ice thickness" and divide by 0.96 to find h_i. Interpolate or extrapolate to determine h_i's that apply to winter peaks and to H_B. Note whether any discharge measurements were made after the time of H_B and before the end of breakup. This would give an indication of the accuracy of discharge and effective stage determinations corresponding to H_m and ΔH_m.

vi) Tabulate pertinent information, including remarks on the accuracy or uncertainty associated with each value.

6.4 Interpretation

No specific rules for interpretation of the data gathered during the processing phase can be formulated at present. Initially, much will depend on the user's own ideas as well as on the type of breakup processes involved, e.g., premature, overmature or intermediate events. A few possibilities are illustrated and discussed elsewhere in this report while it is reasonable to expect that interpretation methods will be continuously updated as understanding increases.

6.5 Presentation of Data

When processing is completed, it would be desirable to issue a summary report. Such reports could then be utilized by various users either for research or site-specific forecasting purposes. It is envisaged that a typical report would include the following information:

i) Brief description of gauge site and ice regime, as outlined in Section 6.2, or indications as to where and how this information could be obtained by the user.

ii) Stage-discharge relation (open-water).

iii) Tables summarizing data on each freeze up-breakup sequence, i.e., H_F, H_B and times of their occurrence; summaries of discharge measurements performed during the sequence - include date, discharge, ice thickness and stage; H_m, ΔH_m and times of their occurrence, along with prevailing discharges - indicate how reliable are the discharge values. As an example, the summary form used by the writer is shown in Table 1.

iv) Data interpretation (optional).

v) Appendices including copies of recorder charts during the freeze up and breakup events; local observer's notes; discharge measurement notes; and applicable meteorological records (optionally) - include temperature, precipitation and bright sunshine.

7.0 SUMMARY AND CONCLUSIONS

A major need for field data pertaining to various aspects of river ice breakup and jamming has been identified. While indications are that ice monitoring activities in Canada will increase in the future, it was suggested that considerable benefits can be derived from exploitation of existing data sources and specifically hydrometric gauge records. Guidelines for extracting pertinent information from such records were presented and associated limitations discussed. Methods for interpreting this information as a means of developing related forecasting procedures were illustrated by means of a case study. At the same time it was shown that no universal interpretation rules can be formulated at present. To improve this situation, many more case studies are needed. In turn, this task can be greatly facilitated by the participation of operating agencies in, at least, the data extraction phase from the hydrometric gauge records. Preparation and issue of pertinent reports with or without the interpretation phase is strongly recommended. Discussion of data extraction methods revealed several instances where modest increases of the gauge operation effort

would result in substantial improvements of the accuracy and utility of the records for ice-related applications.

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Much of the work discussed herein has been a part of a long-term study initiated in 1979 by the Environmental Hydraulics Section, Hydraulics Division, National Water Research Institute, in cooperation with the Guelph Office of Water Survey of Canada. The Nashwaak River example was carried out jointly with the Fredericton Office of Water Survey of Canada, partly under the auspices of the N.B. Sub-Committee on River Ice. Comprehensive reviews and many valuable comments contributed by members of the NRCC Working Group on River Ice Jams are gratefully acknowledged. Reviews by Dr. T.M. Dick and Dr. Y.L. Lau are appreciated.

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APPENDIX - EXAMPLE OF DATA INTERPRETATION

This example is based on the Nashwaak River case study of Beltaos and Lane (1982). The study area is shown in Fig. 6. Details of data extraction and interpretation are given in Beltaos and Lane (1982) while Table 1 summarizers selected parameters extracted from the records.

Initiation of Breakup

The data interpretation has followed Shulyakovskii (1963), after some initial verifications of the basic premises. First, H_B was plotted versus H_F where a trend for H_B to increase with H_F was indicated. However, there was considerable scatter suggesting additional effects. Next, the difference (H_B - H_F) was postulated to depend on h_i and Σq, the total amount of heat input to the ice cover per unit surface area. The latter is an accumulation of daily average heat fluxes (q) until the time of breakup initiation; heat transfer from the water is ignored. Calculation of Σq involves several simplifying assumptions so that Σq must be viewed as a mere index of the true amount of absorbed heat.

At the time of preparing the report (Beltaos and Lane, 1982) very little information on h_i was available, hence (H_B - H_F) was plotted versus Σq, as shown in Figure 7. The scatter is large but the data points appear to follow the anticipated trend. Later, additional ice thickness data were made available to the writer by P. Tang of N.B. Environment which enabled a two-parameter correlation* as follows. First, (H_B - H_F) was plotted versus h_i by noting the value of Σq beside each data point. This indicated both an increase of (H_B -

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* There still remained a few years without any ice thickness measurements. For these years, h_i was estimated using a correlation of measured h_i versus time from H_F. This procedure could involve errors as large as ±30%. Water surface to bottom of ice distances quoted by WSC have been divided by 0.92 to obtain h_i though this is recognized to be a crude approximation.

H_F) with h_i and a decrease with Σq. The upper envelope of the data points, assumed representative of the case $\Sigma q = 0$, could be described by the straight line $H_B - H_F = 2.5\ h_i$. Next, the deviation of any one data point from the upper envelope [$= 2.5\ h_i - (H_B - H_F)$] was computed and plotted versus Σq, as shown in Figure 8. The latter seems to be an improvement over Figure 7; the scatter is reduced while only one event, associated with uncertain record intepretation, does not fit the general trend. The scatter in Figure 8 is still fairly large and this could be partly attributed to: (a) the crudeness of H_B determinations - no local observer's reports were available; and (b) the empiricism utilized in arriving at the final parameter that is plotted versus Σq - lack of a theoretical framework of breakup processes. A favourable circumstance is that even a large error in predicting H_B usually translates to acceptable error in predicting the time of breakup initiation because the corresponding stage gradients with respect to time are large.

Maximum Breakup Stage

As discussed earlier, flow discharge is a major factor influencing H_m. However, discharge data for the Nashwaak River study are uncertain so that the plot of Fig. 9, showing H_m'(=H_m - stage at zero discharge) versus prevailing discharge is merely of qualitative value. It is noted that some of the data points in Fig. 9 represent conditions of maximum ice effect, ΔH_m, in instances where the latter did not coincide with H_m. Also plotted in Fig. 9 is the theoretical relationship between equilibrium jam stage and discharge for comparison (Beltaos 1983a). The latter is seen to provide a satisfactory upper envelope up to a certain discharge, but to consistently overpredict the stage beyond this discharge. This is a typical trend, reflecting the fact that increasing discharge reduces the probability of equilibrium jam formation (Beltaos 1983a). For practical purposes, an upper envelope of the data points could be drawn and used to forecast

potential H_m values. Whether and how closely the potential H_m is to be realized in a given season depends on the number and stability of ice jams that form near the gauge site, as discussed earlier. In turn, such effects are controlled by channel and floodplain configuration as well as the competence of the ice cover during breakup. The former factor is difficult to assess at present because the behaviour of ice jams is unknown once the bankfull stage is exceeded (see also Calkins 1983). On the other hand, experience suggests that the competence of an ice cover should be an important factor influencing H_m and this possibility is considered next.

Since the competence of an ice cover can be defined in terms of its strength, thickness and width, it may be of interest to explore $\sum q$, h_i and H_F (rough measure of ice cover width) as possible factors influencing H_m. Figure 10 shows H_m plotted versus H_F. The data points define an upper envelope which increases with H_F. The deviation of the observed value of H_m from the corrsponding upper envelope value is plotted versus $\sum q$ in Figure 11. This results in another upper envelope which confirms the anticipated trend. It thus appears that H_F and $\sum q$ define an upper limit for, or potential, H_m*. Whether and how closely this potential will be realized in any one breakup event, depends on a number of other factors, e.g., discharge, local jamming conditions, etc.

Frequency of Occurrence of H_m

A simple frequency analysis on H_m values was also performed using the following equation (Gerard and Karpuk, 1979):

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*Notes: (1) One would expect that h_i should also be relevant here but this possibility has not been investigated in this example. Effects of freeze up conditions and h_i on H_m have also been discerned at other sites (British Columbia Hydro 1975; Beltaos 1983b).

(2) Strictly speaking, $\sum q$ should be calculated to the time of H_m. However, the value used in Figure 5 applies to the time of H_B. This was thought sufficient given that the Nashwaak River study was exploratory. Nevertheless, both these elaborations should be kept in mind for future work.

$$P = (m - \frac{3}{8})/(N + \frac{1}{4}) \quad (1)$$

in which P = probability of a given stage being equalled or exceeded during any one breakup event; m = rank; and N = total number of analyzed breakup events. The fact that, occasionally, there have been two breakup events in the same season has been ignored and all events have been assumed to be independent so as to increase the value of N. This may or may not be valid but more data are required to clarify this point.

For convenience of plotting, the gauge height has been reduced by H_o, the value corresponding to zero discharge, i.e., use has been made of $H_m' = H_m - H_o$. In this manner, the event $H_m' \geq 0$ has a probability P = 1. After performing the frequency analysis, H_m' can be plotted versus P on various types of probability charts as a means of exploring the mathematical form of the H_m' distribution. Gerard and Karpuk (1979) suggested that the log-normal distribution is a possible candidate in this respect and found a linear relationship on plotting their data on log-normal probability paper. The Nashwaak River data are compared with Gerardand Karpuk's Peace River data in Fig. 12 where the ratio $H_m'/H_m'_{50}$ is plotted versus P. Note that $H_m'_{50} = H_m'$ at P = 0.5. Fig. 12 shows that a linear fit could only partly describe the Nashwaak River results. It is also noted that there is a near coincidence with the Peace River data in the linear range ($0.1 \leq P \leq 0.90$) but the reason for this is not known.

Limitation

Beltaos and Lane (1982) emphasized that the Nashwaak River study was purely empirical and site-specific. Thus, extrapolation of their findings to other sites or to meteorological conditions not representative of the range covered by the years of record, would not be justifiable. Clearly, a generalization of forecasting methods on river ice breakup requires many more case studies of this kind. Only then

would it become possible to examine additional effects such as stream geomorphology, basin characteristics and climatic conditions.

Example of Application

Suppose that, in a given season, H_F = 2.50 m. On March 14, a warm weather trend and increased runoff are forecast for the next few days, as indicated below:

	Date in March	Mean Daily Air Temperature (°C)	Hours of Sunshine	Gauge Height (m)	Discharge (m^3/s)
Known:	13	-5	0	1.52	33
	14	-2	4	1.52	33
Forecast:	15	2	5	1.83	48
	16	4	8	2.44	86
	17	6	10	3.35	155
	18	Turning cold			155

From previous ice thickness measurements, h_i is fixed at 0.40 m. To calculate Σq, we can use the following equation (Beltaos and Lane, 1982):

$$q = a\theta + bS_u + c \qquad (2)$$

in which = daily amount of heat (J/cm^2); θ = daily average air temperature (°C); S_u = hours of bright sunshine; and a, b, c are coefficients which depend on latitude and time of year. For the Nashwaak River at Durham Bridge at about mid-March, the values of a, b

and c are 68.7, 26.8 and 160.1, respectively. With these, we compute q = negative on March 13; and 1 = 130, 432, 649 and 840 for the subsequent four days. Correspondingly, values of Σq and $H_{\text{ç}B}$ (via Figure 8) are as follows:

Date in March	Σq (J/cm^2)	H_B (m)	Gauge Height (m)
13	nil	3.50	1.52
14	130	3.40	1.52
15	562	2.97	1.83
16	1211	2.46	2.44
17	2051	2.05	3.35

Assuming that the above values apply at about noon of each day, breakup is expected to start sometime on March 16, this being the first date on which H_B will become less than the expected gauge height. The corresponding value of Σq is 1211 J/cm^2. Figure 11 then gives H_m - 1.22 - 1.88 $H_F \leq$ -0.10 m. It follows that H_m should not exceed 4.1 m. If the peak instantaneous discharge is assumed to be equal to 155 m^3/s, then Figure 9 would give $H_m \leq$ 4.4 m, so that the former limit (4.1 m) would apply. If, on the other hand, the discharge were to peak at, say, 80 m^3/s, Figure 9 wuld give $H_m \leq$ 3.7 m and this could be taken as the applicable limit of H_m.

TABLE 1. EXAMPLE OF DATA SUMMARY FORM

GAUGE SITE _____ SEASON _____
EVENT NO. ___ OF ___ . PROCESSED BY _____ ON _____

BEGIN ICE EFFECT ON STAGE AT _____ h ON _____
STAGE = _____ m. DISCHARGE = Q = _____ m^3/s.

H_F = _____ m ON _____

"WINTER" PEAKS

H_B> (m)	TIME/DATE	h_i (cm)	M/E [1]	S_T (°C-days)	$\sum q$ (J/cm^2)	(other)

DISCHARGE MEASUREMENTS

TIME/DATE	Q (m^3/s)	STAGE (m)	EFFECTIVE STAGE (m)	h_i (cm)	ICE CONDITIONS

BEGIN RISE TO BREAKUP AT _____ h ON _____
STAGE = _____ m. DISCHARGE = Q = _____ m^3/s.

H_B (m)	TIME/DATE	h_i (cm)	M/E [1]	S_T (°C-days)	$\sum q$ (J/cm^2)	(other)

H_m (m)	TIME/DATE	Q (m^3/s)	ΔH_m (m)	TIME/DATE	STAGE (m)	Q (m^3/s)

REMARKS ON RELIABILITY OF H_F, H_B, H_M AND Q VALUES:

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(1) Use M for h_i determinations based on interpolations or extrapolations of one or more measurements; use E for estimates based on other years' experience.

TABLE 2. SUMMARY OF BREAKUP CHARACTERISTICS AND ASSOCIATED PARAMETERS; NASHWAAK R. AT DURHAM BRIDGE 1965-81

Season	Winter Peaks			Breakup					Ice Thickness		Maximum Stage			Maximum Ice Effect				Remarks
	H_F (m)	Time of H_F (day/month)	H (m)	Time of H	Σq (J/cm²)	Range of H_B (m)	Probable H_B (m)	Time of H_B	Σq (J/cm²)	Σq Calculated Σ_0 (day/mo.)	Time of h_i Measurement (m)		H_m (m)	Time of H_m	Discharge (m^3/s)	ΔH_m (m)	Time of ΔH_m	H (m)	Discharge (m^3/s)	
1964-65	2.56	13/1	2.20	7/3	2263															Breakup undefinable
1965-66	1.40	29/12					1.58	22/3	2807	22/3			2.23	26/3	83	0.55	22/3	1.59	22	
1966-67	1.80	27/12					1.71	7/4	5992	7/4	0.70	16/3	1.73	11/4	15	0.82	11/4	1.73	15	
1967-68	2.34	1/1					3.25	4/2	461	4/2	0.37	12/1	3.87	4/2	100	2.05	4/2	3.87	100	h_i not representative of breakup value
1967-68	3.35	6/2	2.78	25/3	3855		2.71	30/3	6411	30/3			3.40	1/4	93	1.65	1/4	3.40	93	
1968-69	2.25	29/12	1.84	27/1	0		1.87	28/3	3562	28/3			1.87	30/3	22	0.82	30/3	1.87	22	Difficult to define breakup
1969-70	3.44	1/1				1.98 - 5.31		2-4/2	587	3/2			5.31	4/2	697	0.92	4/2	5.31	697	- " -
1969-70	2.29	16/2				1.43 - 1.80		27/3	4819	27/3			2.05	28/3	34	0.82	28/3	2.05	34	- " -
1970-71	1.65	10/12	2.07	15/2	251	0.90 - 1.31		3-4/4	6872	3/4	0.64	11/3	1.96	14/4	49	0.93	6/4	1.79	30	
			1.83	18/3	2179				7542	4/4										
1971-72	1.19	7/12	1.34	13/12	251	1.62 - 2.19		17-19/3	1048	17/3	0.73	6/3	2.49	20/3	28	1.39	20/3	2.49	28	
			1.43	16/1	0				1467	18/3										
			1.59	16/2	1383															
			1.74	12/3	0															
1972-73	2.72	14/12	2.79	5/2	0	1.91 - 2.35		17-18/3	5363	18/3			2.61	19/3	76	1.27	18/3	2.53	40	
			2.16	13/3	3352															
1973-74	2.20	10/1	2.22	8/3	3017	1.56 - 2.19		4-7/4	2682	4/4	0.70	27/3	2.19	7/4	56	0.76	7/4	2.19	56	
									4735	7/4										
1974-75	2.19	4/1	2.15	22/3	2137	1.48 - 1.68		4-8/4	4483	4/4			1.68	8/4	32	0.52	8/4	1.68	32	Difficult to define breakup
									6034	8/4										
1975-76	2.44	14/12	3.38	29/1	277		2.19	27/3	4064	26&27/3(avg)	0.73	2/3	2.94	29/3	126	1.15	30/3	2.78	126	
1976-77	1.89	12/12	2.69	27/12	0	1.49 - 1.53		27-28/3	6411	27/3			2.29	31/3	74	0.69	31/3	2.29	74	Difficult to define H_F and H_B; H_F= 2.29 on 29/12 might be better
			1.91	16/3	3645															
1977-78	1.82	11/12				2.15 - 2.51		26/12	21	26/12			2.51	26/12	42	1.23	26/12	2.51	42	Probably thin ice
1977-78	2.74	18/1				1.42 - 1.54		11/4	7123	11/4			1.76	15/4	47	0.44	15/4	1.76	47	Difficult to define H_F
1978-79	1.10	27/11				< 1.93		3/1	293	2/1			2.57	3/1	74	0.96	3/1	2.57	74	
1978-79	3.40	7/1	3.26	31/1	1089	1.54 - 2.85		5-8/3	922	5/3			3.20	26/3	207	1.15	15/3	2.76	74	Difficult to define breakup
									2179	7/3										
1979-80	2.25	4/1				2.71 - 3.01		18-19/3	629	18/3	0.55	13/3	3.03	19/3	105	1.65	1/4	2.84	36	
1980-81	2.41	17/12				1.81 - 2.11		19-23/2	712	18/2			2.12	23/2	68	0.57	23/2	2.12	68	
									2556	22/2										

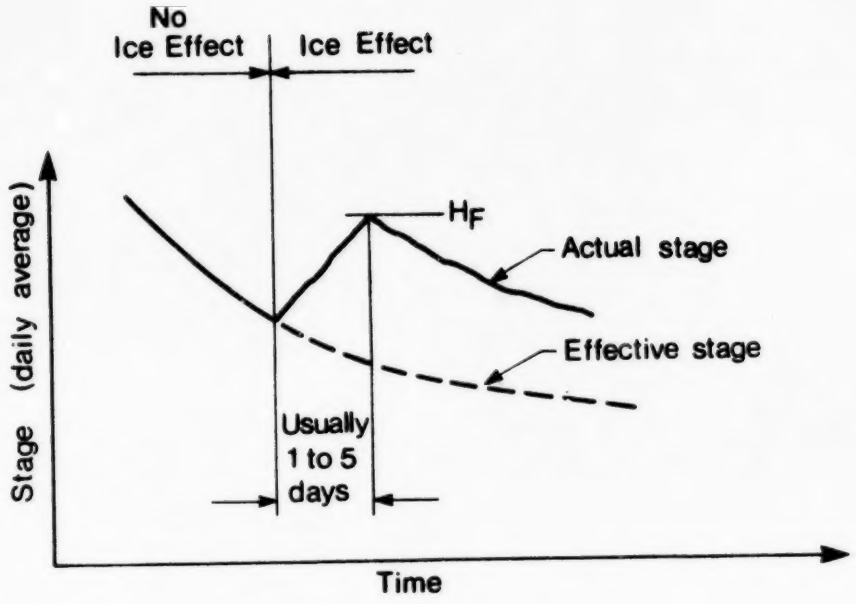

Fig. 1. Schematic illustration of daily stage variation with time during beginning of freeze up.

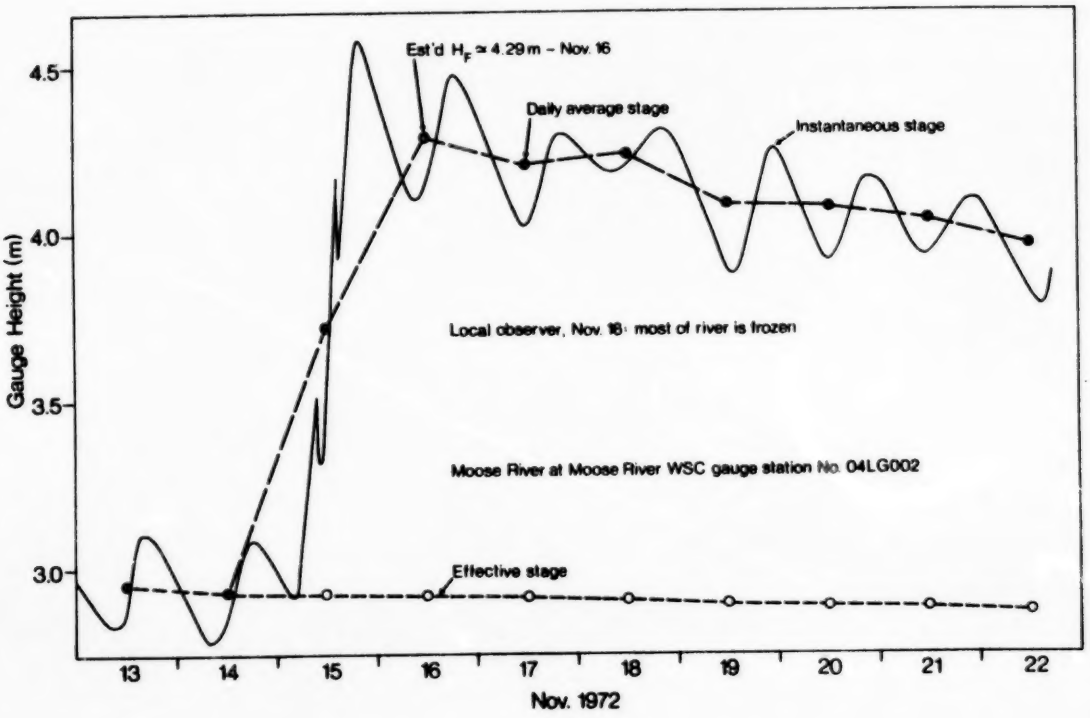

Fig. 2. Freeze up stage record, Moose River at Moose River.

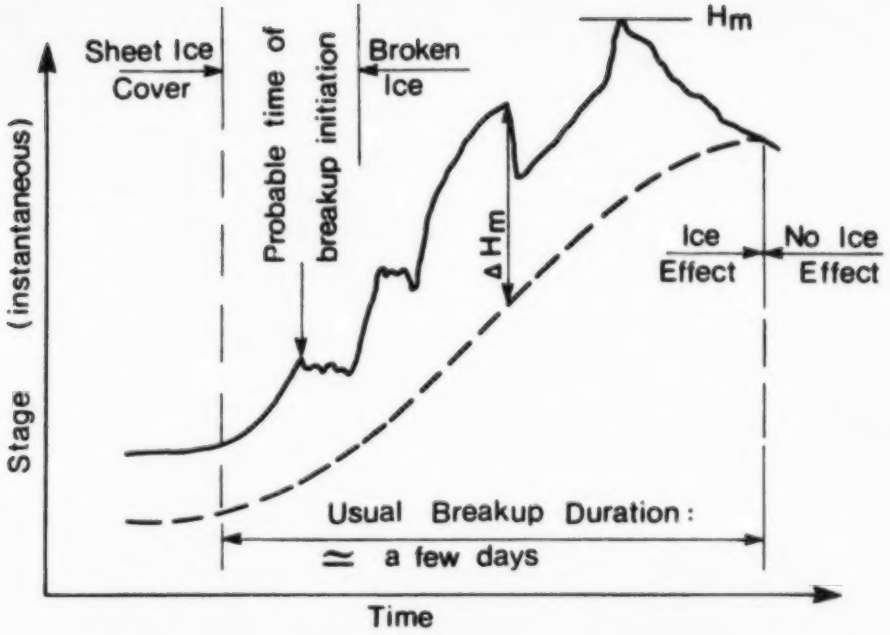

Fig. 3. Schematic illustration of instantaneous stage variation with time during breakup.

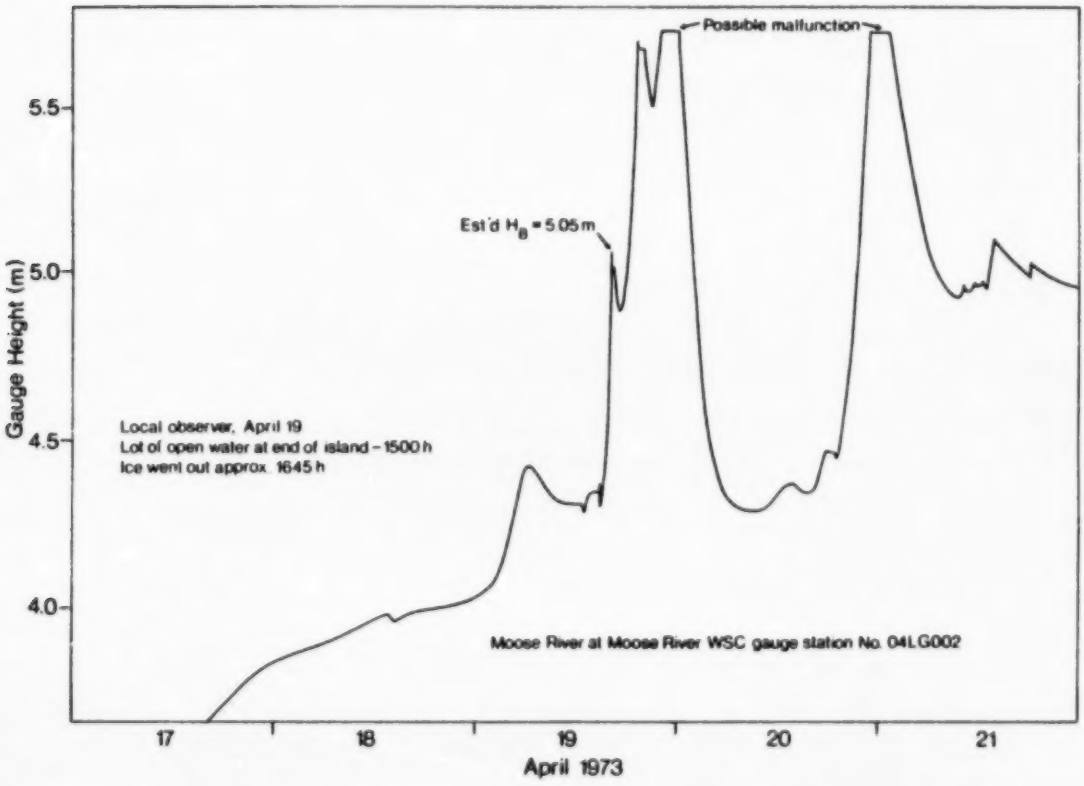

Fig. 4. Breakup stage record, Moose River at Moose River.

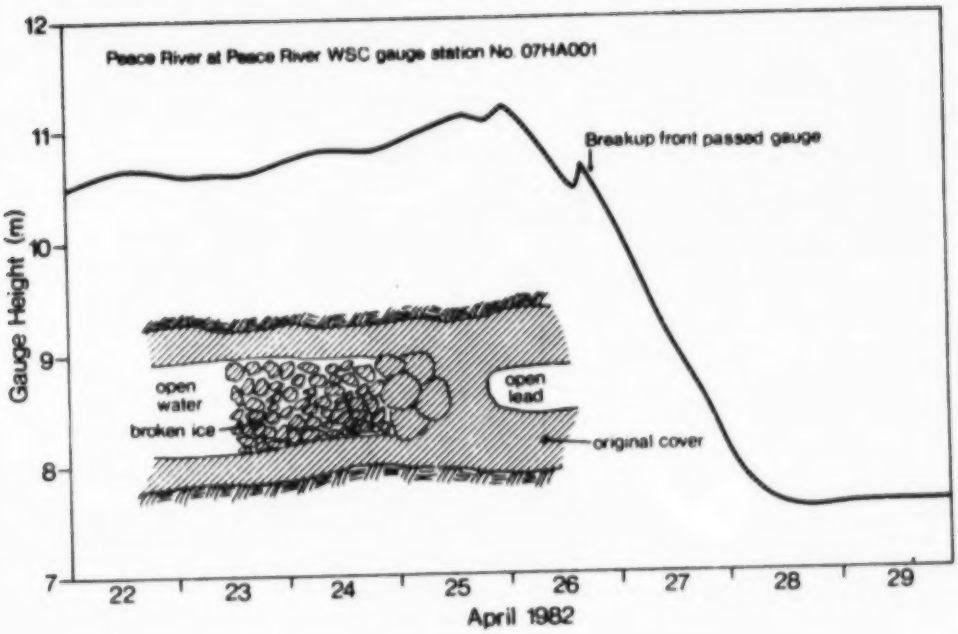

Fig. 5. Stage record during overmature breakup (stage data taken from Fonstad, 1982).

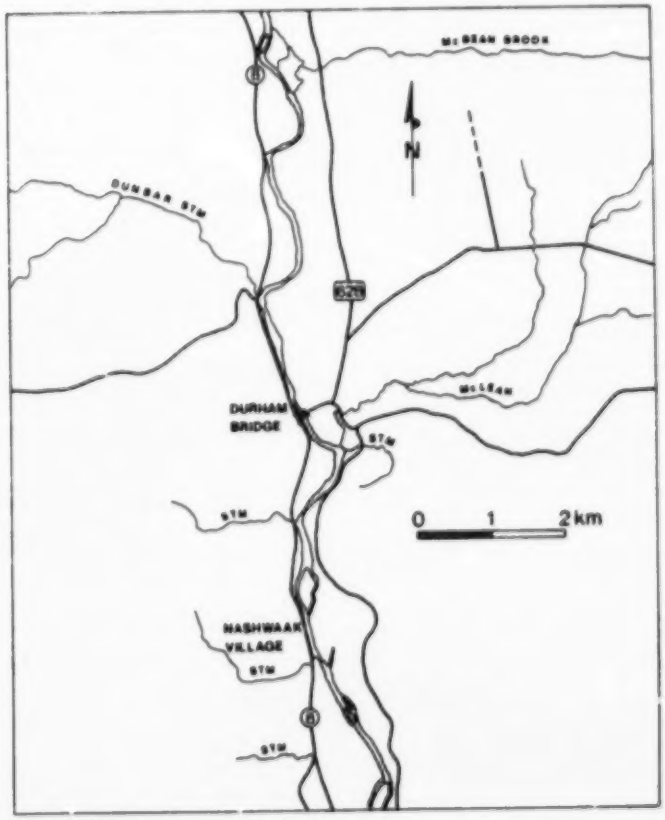

Fig. 6. Plan view of Nashwaak River near Durham Bridge.

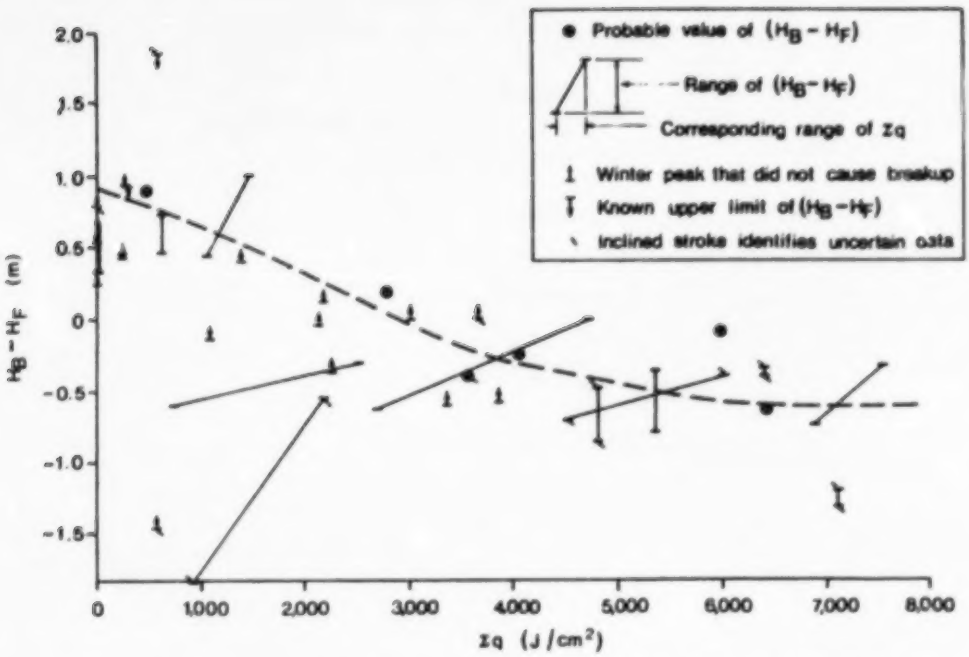
Probable value of $(H_B - H_F)$
Range of $(H_B - H_F)$
Corresponding range of Σq
Winter peak that did not cause breakup
Known upper limit of $(H_B - H_F)$
Inclined stroke identifies uncertain data
$H_B - H_F$ (m)
Σq (J/cm²)

Fig. 7. Stage rise above H_F required to initiate breakup, as a function of Σq.

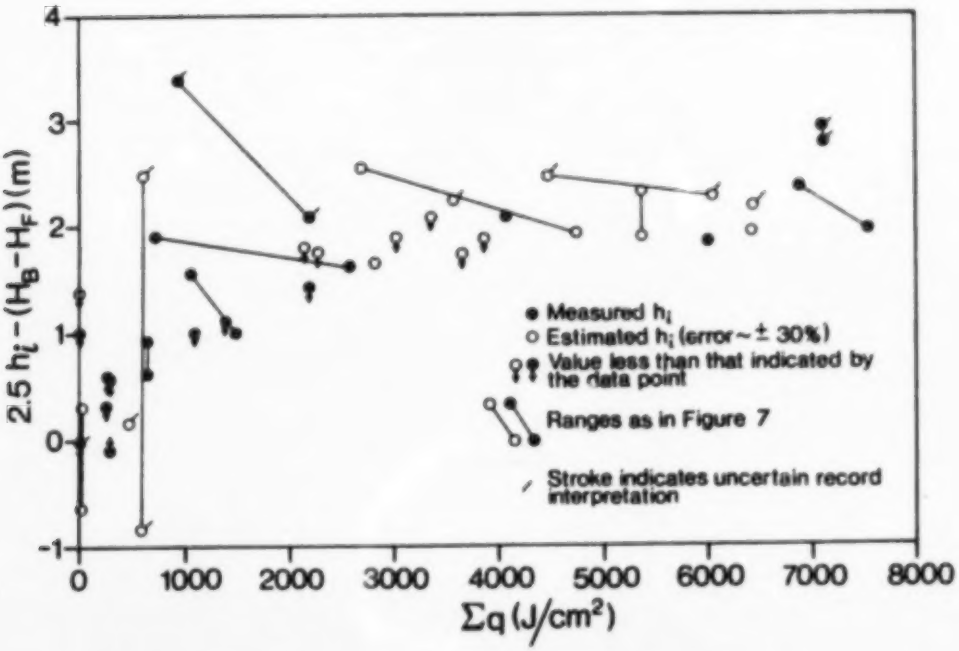
Measured h_i
Estimated h_i (error ~ ± 30%)
Value less than that indicated by the data point
Ranges as in Figure 7
Stroke indicates uncertain record interpretation
$2.5\ h_i - (H_B - H_F)$ (m)
Σq (J/cm²)

Fig. 8. Variation of $2.5\ h_i - (H_B - H_F)$ with Σq.

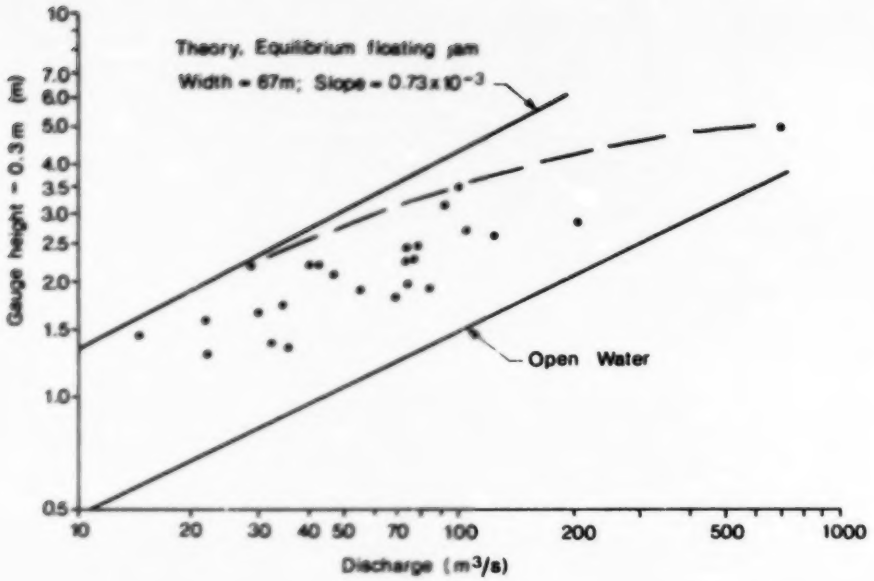

Fig. 9. Effect of discharge on stage during breakup.

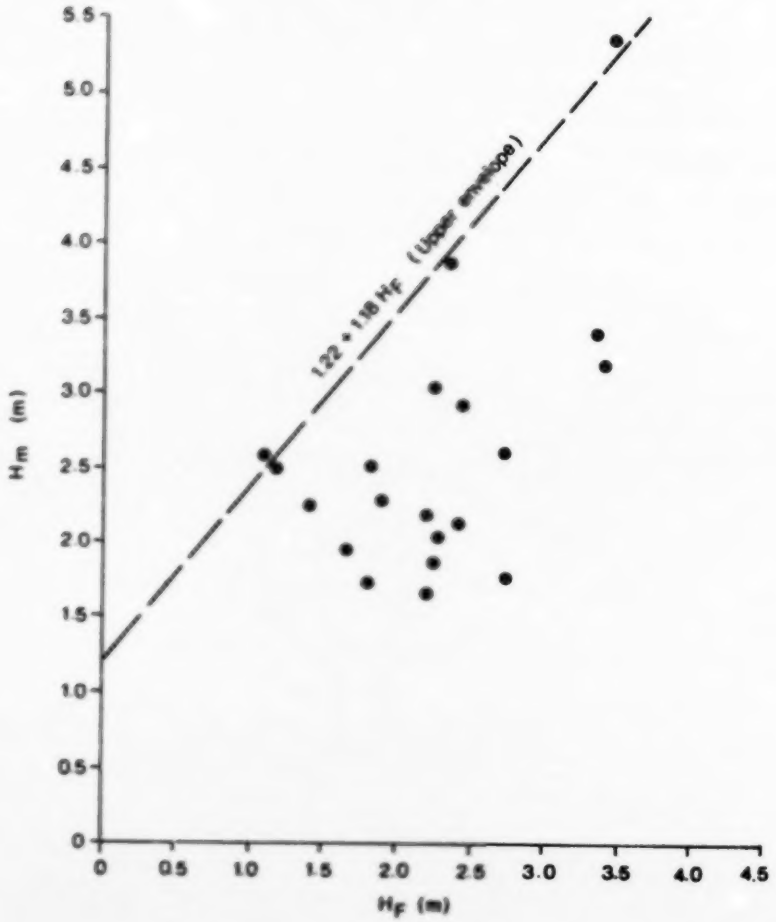

Fig. 10. Maximum stage during breakup versus H_F.

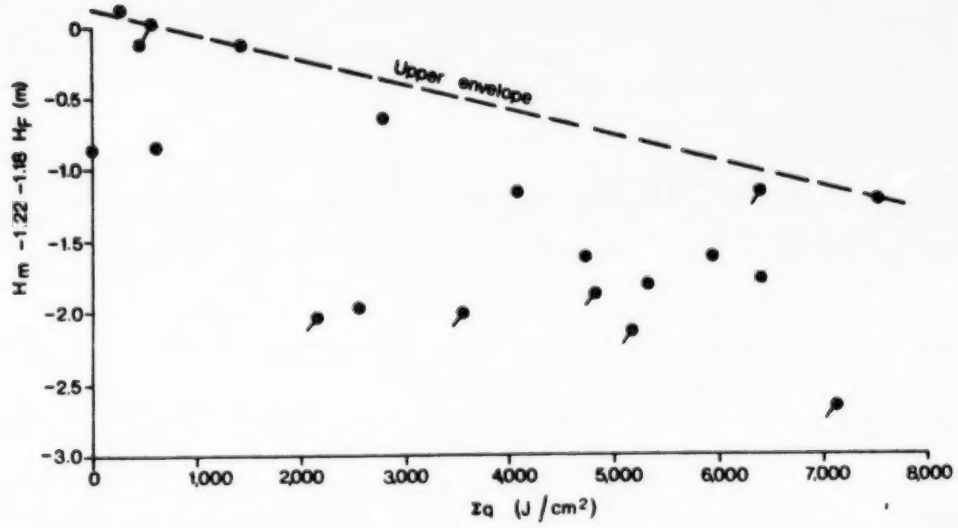

Fig. 11. Effect of Σq on H_m.

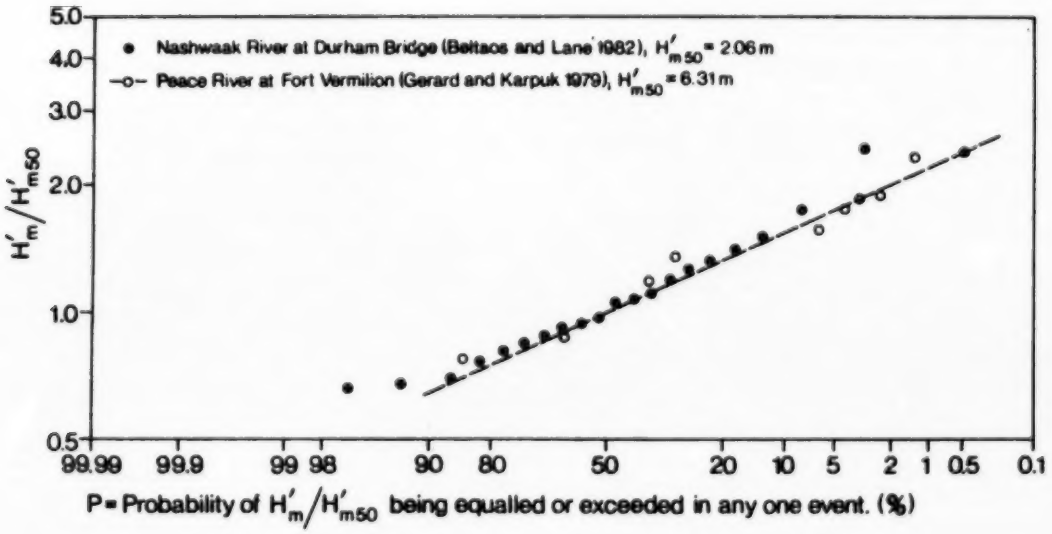

Fig. 12. Results of probability analysis.